This set contains three pages (beginning with this page)
All questions must be answered
Questions 1 and 2 each weigh 25 % while question 3 weighs 50 %. These weights, however, are only indicative for the overall evaluation.

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Spring 2010

MONETARY ECONOMICS: MACRO ASPECTS
SOLUTIONS TO JUNE 16 EXAM

QUESTION 1:

Evaluate whether the following statements are true or false. Explain your answers.

(i) In the simple New-Keynesian model, optimal stabilization of inflation shocks requires that the central bank can credibly commit not to affect private-sector expectations.

A False. The central idea in the New-Keynesian model is how macroeconomic outcomes are determined by expectations about the future. Being able to affect those expectations is therefore an advantage that is exploited in a commitment solution.

(ii) Under strict inflation targeting, in the Svensson (1997) sense, the optimal nominal interest-rate rule should only depend on inflation.

A False. In the model, current output has predictive contents for future inflation, and therefore inflation expectations. Even if the objective is only to stabilize inflation, or, equivalently inflation expectations, it is therefore optimal to respond to output movements. Output has the function of an intermediate target in the model.

(iii) When an explicit shopping-time motive is introduced as a way of indirectly having money in the utility function, the Friedman rule is never optimal.

A False. In the shopping-time model, a positive interest rate is an opportunity cost of holding money, and thus an opportunity cost on saved shopping time. It therefore creates a distortion in the shopping-time choice, which is eliminated with a zero nominal interest rate.

QUESTION 2:

Monetary policy and an "output target conservative" central banker

Consider the following log-linear AS curve in a closed economy:

$$y_t = a\left(\pi_t - \mathrm{E}_{t-1}\pi_t\right) + \varepsilon_t, \qquad a > 0, \tag{1}$$

where y_t is output, π_t is inflation and ε_t is a mean-zero, serially uncorrelated shock. E_{t-1} is the rational expectations operator conditional upon all information up to and including period $t-1$. For simplicity π_t is taken to be the instrument of monetary policy. The aim of policy is to minimize

$$V = \frac{1}{2}\sum_{t=0}^{\infty}\beta^t\left[\lambda\left(y_t - k\right)^2 + \pi_t^2\right], \qquad k > 0, \quad \lambda > 0, \quad 0 < \beta < 1. \tag{2}$$

(i) Discuss (1) and (2) with focus on the underlying economic mechanisms, and derive the optimal time-consistent outcomes for output and inflation. What is the inefficiency of the solution? Explain.

A (1) is the aggregate supply curve, which can be derived from the Lucas misperceptions model or a model with one-period sticky wages. Inflation above the anticipated will increase output. Either due to the aggregate vs. local confusion in a Lucas set up, or due to the erosion of real wages in a set up with nominal wage rigidity. (2) is the social loss which shows that output and inflation deviations from k and zero, respectively, are considered harmful. $k > 0$ implies that the natural rate of output $\mathrm{E}[y] = 0$ is considered sub-optimal due to imperfections in the economy.

The time-consistent outcomes for output and inflation follow from the first-order condition of the policymaker (insert y_t from (1) into (2), differentiate w.r.t. π_t taking ε_t and $\mathrm{E}_{t-1}\pi_t$ as given, and set equal to zero):

$$\lambda a \left(a \left(\pi_t - \mathrm{E}_{t-1}\pi_t\right) + \varepsilon_t - k\right) + \pi_t = 0.$$

Take period-$t-1$ expectations on both sides:

$$\begin{aligned} \mathrm{E}_{t-1}\lambda a \left(a \left(\pi_t - \mathrm{E}_{t-1}\pi_t\right) + \varepsilon_t - k\right) + \mathrm{E}_{t-1}\pi_t &= 0, \\ -\lambda a k + \mathrm{E}_{t-1}\pi_t &= 0, \end{aligned}$$

and thus

$$\mathrm{E}_{t-1}\pi_t = \lambda a k.$$

Using the solution for inflation expectations in the first-order condition gives

$$\lambda a \left(a \left(\pi_t - \lambda a k\right) + \varepsilon_t - k\right) + \pi_t = 0,$$

from which time-consistent inflation follows as

$$\pi_t = \lambda a k - \frac{\lambda a}{1 + \lambda a^2}\varepsilon_t.$$

Output follows by inserting this (and $\mathrm{E}_{t-1}\pi_t = \lambda a k$) into (1):

$$\begin{aligned} y_t &= a\left(\lambda a k - \frac{\lambda a}{1+\lambda a^2}\varepsilon_t - \lambda a k\right) + \varepsilon_t, \\ y_t &= \frac{1}{1+\lambda a^2}\varepsilon_t. \end{aligned}$$

The inefficiency of this solution is that inflation is excessively high; i.e., there is a positive inflation bias. This arises due to $k > 0$, as this gives the policymaker an incentive to expand in order to increase output above the natural rate. This, however, is anticipated by the private sector who accordingly sets inflation expectations so high that the policymaker validates them (increasing inflation would be too harmful in terms of inflation). The supply shock, however, is efficiently stabilized.

(ii) Society now delegates monetary policymaking to a central banker who aims at an output level lower than the social optimal. Hence, it has a loss function given by

$$V^c = \frac{1}{2}\sum_{t=0}^{\infty}\beta^t\left[\lambda\left(y_t - k^c\right)^2 + \pi_t^2\right], \qquad k^c < k. \tag{3}$$

Show formally how the time-consistent outcomes change relative to those derived in (i), and assess whether delegation of this form will always be beneficial.

A The solution procedure is the exact same as under (i) (no derivations are in fact needed here), and the time-consistent outcomes for inflation and output become

$$\begin{aligned} \pi_t &= \lambda a k^c - \frac{\lambda a}{1+\lambda a^2}\varepsilon_t, \\ y_t &= \frac{1}{1+\lambda a^2}\varepsilon_t. \end{aligned}$$

This form of delegation is always beneficial, as a lower value of k^c does not alter stabilization of shocks, but "only" lowers the inflation bias. By setting $k^c = 0$, the bias can be completely eliminated at no stabilization cost; i.e., the output goal should be the natural rate of output. The economy will then be in the optimal equilibrium. (It is excellent to mention that in comparison with a Rogoff-conservative central banker, there is a trade-off between reducing the inflation bias and worsening the stabilization performance.)

(iii) Discuss the relationship between this form of delegation and Walsh's linear inflation contract.

A Walsh's linear inflation contract induces a constant marginal penalty of inflation, which—appropriately designed—eliminates the inflation bias, while leaving stabilization performance unaltered.[1] The delegation to a central banker with an appropriate output goal is thus an equivalent way of shaping the policymaker's incentives such that the best possible economic outcomes are achieved.

[1] This is not to be expected of the student, but in terms of the first-order condition, it will become

$$\lambda a \left(a\left(\pi_t - \mathrm{E}_{t-1}\pi_t\right) + \varepsilon_t - k\right) + \pi_t + MP = 0$$

under a Walsh contract, where $MP > 0$ is the contract parameter in the linear contract $t\left(\pi_t\right) = t_0 - MP\pi_t$. Having $MP = \lambda ak$ will eliminate the term with k without affecting the response to the shock.

QUESTION 3:

Cash and credit goods and monetary policy

Assume a model of a closed economy formulated in discrete time, where representative individuals have utility functions

$$U = \sum_{t=0}^{\infty} \beta^t \left[u(c_t) + w(d_t) \right], \qquad 0 < \beta < 1, \tag{1}$$

and budget constraints

$$f(k_{t-1}) + \tau_t + (1-\delta) k_{t-1} + \frac{1}{1+\pi_t} m_{t-1} = c_t + d_t + k_t + m_t, \tag{2}$$

where c_t is consumption of "cash goods", d_t is consumption of "credit goods," m_t is real money balances at the end of period t, k_{t-1} is physical capital at the end of period $t-1$, τ_t are monetary transfers by the government, $0 < \delta < 1$ is capital's rate of depreciation and π_t is the inflation rate. The functions u, w and f are increasing and strictly concave.

Purchases of cash goods are subject to a cash-in-advance constraint, which is modelled by the constraint

$$c_t \leq \tau_t + \frac{1}{1+\pi_t} m_{t-1} \equiv a_t. \tag{3}$$

(i) Discuss the model given by (1), (2) and (3).

A (1) shows that we consider a representative, infinitely lived individual who obtains utility from two different types of consumption goods, c and d. (Hence, a labour supply decision is not present in the model.) (2) is the budget constraint, where the left-hand side is available resources in period t composed of income, government monetary transfers, the value of the capital stock net of depreciation and the real value of money balances carried over from last period. The right-hand side shows that these resources can be used on the consumption goods and the two types of assets in the economy: physical capital and real money. Equation (3) is a cash-in-advance constraint stating that consumption of the good c in period t cannot exceed the period's monetary resources. It is noteworthy that the good d is not subject to this constraint.

(ii) Derive the relevant first-order conditions for optimal individual behavior, For this purpose, use the value function

$$V\left(k_{t-1}, a_t\right) = \max\left\{u\left(c_t\right) + w\left(d_t\right) + \beta V\left(k_t, a_{t+1}\right) - \mu_t\left(c_t - \tau_t - \frac{1}{1+\pi_t}m_{t-1}\right)\right\},$$

where μ_t is the multiplier on (3), and where the maximization is over c_t, d_t, m_t and k_t and subject to (2). [Hint: Simplify the problem by using (2) to substitute out k_t in the value function]

A Using the hint, one finds the following three first-order conditions:

$$\begin{aligned} u'\left(c_t\right) - \beta V_k\left(k_t, a_{t+1}\right) - \mu_t &= 0, && (*) \\ w'\left(d_t\right) - \beta V_k\left(k_t, a_{t+1}\right) &= 0, && (**) \\ -\beta V_k\left(k_t, a_{t+1}\right) + \frac{1}{1+\pi_{t+1}}\beta V_a\left(k_t, a_{t+1}\right) &= 0. && (***) \end{aligned}$$

(iii) Interpret the first-order conditions and show that they (along with the expressions for the partial derivatives of the value function derived using the Envelope Theorem) can be combined into the following system:

$$\begin{aligned} \frac{u'\left(c_t\right)}{w'\left(d_t\right)} &= \frac{\lambda_t + \mu_t}{\lambda_t}, \\ \lambda_t &= \frac{\beta}{1+\pi_{t+1}} V_a\left(k_t, a_{t+1}\right), \\ \beta^{-1}\lambda_{t-1} &= \lambda_t R_{t-1}, \\ V_a\left(k_{t-1}, a_t\right) &= \lambda_t + \mu_t, \end{aligned}$$

where $R_{t-1} \equiv 1 + f'\left(k_{t-1}\right) - \delta$ is the gross real interest rate and $\lambda_t \equiv \beta V_k\left(k_t, a_{t+1}\right)$.

A The interpretation of the first-order conditions are: (*) shows that consumption of c is chosen so the marginal gain in terms of current utility is equalized to the marginal loss which is due to the value loss of less next-period capital and the "liquidity cost" of having to obey the cash-in-advance constraint (as captured by μ_t). (**) is the analogous expression for the choice of consumption of d except that this does not involves a liquidity cost. (***) shows that real money balances are chosen such that the marginal gain in terms of higher monetary wealth in next period is balanced against the marginal cost in terms of less capital. Note that higher inflation reduce the marginal gain.

The envelope theorem gives the following expressions (as all effects from k_{t-1} and a_t on d_t, c_t and m_t can be ignored):

$$V_k(k_{t-1}, a_t) = [f'(k_{t-1}) + 1 - \delta]\, \beta V_k(k_t, a_{t+1}), \qquad (****)$$

$$V_a(k_{t-1}, a_t) = \beta V_k(k_t, a_{t+1}) + \mu_t. \qquad (*****)$$

Now derive the four equations. 1) The first follows by using the definition $\lambda_t \equiv \beta V_k(k_t, a_{t+1})$ together with (*) and (**). 2) The second follows by using the definition $\lambda_t \equiv \beta V_k(k_t, a_{t+1})$ together with (***). 3) The third follows by using the definitions $\lambda_t \equiv \beta V_k(k_t, a_{t+1})$ and $R_{t-1} \equiv 1 + f'(k_{t-1}) - \delta$ together with (****). 4) The fourth follows by using the definition $\lambda_t \equiv \beta V_k(k_t, a_{t+1})$ together with (*****).

(iv) Show whether different monetary policies (here, different inflation rates) have steady-state effects on output, $y^{ss} = f(k^{ss})$. Explain.

A (****) shows that in steady state,

$$1 = [f'(k^{ss}) + 1 - \delta]\, \beta,$$

which uniquely determines steady-state capital and therefore steady-state output independent of inflation. The reason is that the optimal savings decision is not affected by monetary factors in the model: The dynamics of the marginal value of capital is not affected by inflation.

(v) Is monetary policy steady-state superneutral in the sense that no real variables are affected by inflation? Assess this formally and explain. [Hint: For this purpose, show that $\lambda_t = \beta \frac{\lambda_{t+1} + \mu_{t+1}}{1 + \pi_{t+1}}$.]

A No. Use the hint in steady state,

$$\lambda^{ss} = \beta \frac{\lambda^{ss} + \mu^{ss}}{1 + \pi^{ss}}.$$

(It can be derived by combination of second and fourth equation from (iii).). This can be rewritten as

$$\frac{1 + \pi^{ss}}{\beta} = \frac{\lambda^{ss} + \mu^{ss}}{\lambda^{ss}}.$$

Combined with the first equation of (iii) one then gets

$$\frac{u'(c^{ss})}{w'(d^{ss})} = \frac{\lambda^{ss} + \mu^{ss}}{\lambda^{ss}} = \frac{1 + \pi^{ss}}{\beta}.$$

Hence, inflation affects the *composition* of output by altering the marginal rate of transformation between cash and credit goods. It works as a *tax on cash goods*. Higher inflation will therefore lead to a switch away from the cash good towards the credit good. The intuition is that higher inflation increases the steady-state nominal interest rate (the real interest rate, R^{ss}, is independent of monetary factors), which increases the marginal loss of cash goods relative to credit goods. The perfect answer will acknowledge that the optimal monetary policy is one where the marginal rate of transformation between consumption goods is one, i.e., where

$$\begin{aligned} \frac{1 + \pi^{ss}}{\beta} &= 0, \\ \pi^{ss} &= \beta - 1 < 0 \end{aligned}$$

implying a zero nominal interest rate and deflation according to the Friedman rule.